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**Level 3 – AS91581 – 4 Credits – Internal**

**Investigate Bivariate Measurement Data**

**Written by Jake Wills – MathsNZ –** [**jwills@mathsnz.com**](mailto:jwills@mathsnz.com)

|  |  |  |
| --- | --- | --- |
| **Achievement** | **Achievement with Merit** | **Achievement with Excellence** |
| Investigate bivariate measurement data. | Investigate bivariate measurement data, with justification. | Investigate bivariate measurement data, with statistical insight. |

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**Part 1a: Graphs without Trend Lines**

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| --- | --- |
| C:\CA825085\2CE2AD46-5EBC-49CC-9EA8-1E55C7ACFC2C_files\image003.png | C:\CA825085\2CE2AD46-5EBC-49CC-9EA8-1E55C7ACFC2C_files\image004.png |
| C:\CA825085\2CE2AD46-5EBC-49CC-9EA8-1E55C7ACFC2C_files\image005.png | C:\CA825085\2CE2AD46-5EBC-49CC-9EA8-1E55C7ACFC2C_files\image006.png |
| C:\CA825085\2CE2AD46-5EBC-49CC-9EA8-1E55C7ACFC2C_files\image007.png | C:\CA825085\2CE2AD46-5EBC-49CC-9EA8-1E55C7ACFC2C_files\image008.png |

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**Part 1b: Graphs with Trend Lines**

|  |  |
| --- | --- |
| C:\CA825085\2CE2AD46-5EBC-49CC-9EA8-1E55C7ACFC2C_files\image009.png | C:\CA825085\2CE2AD46-5EBC-49CC-9EA8-1E55C7ACFC2C_files\image010.png |
| C:\CA825085\2CE2AD46-5EBC-49CC-9EA8-1E55C7ACFC2C_files\image011.png | C:\CA825085\2CE2AD46-5EBC-49CC-9EA8-1E55C7ACFC2C_files\image012.png |
| C:\CA825085\2CE2AD46-5EBC-49CC-9EA8-1E55C7ACFC2C_files\image013.png | C:\CA825085\2CE2AD46-5EBC-49CC-9EA8-1E55C7ACFC2C_files\image014.png |

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**Part 2: Problem**

For each of the graphs below write a good relationship question. A question should have:

* Why you are interested in looking at this relationship (i.e. the context).
* What you are trying to find a relationship between.
* What you are planning on predicting (what is on the y-axis).

You should be referencing your context (i.e. research – it doesn’t matter how you reference, just that you do… using footnotes is normally easiest). The first one has been done for you.

|  |  |
| --- | --- |
| Machine generated alternative text: 40  E  35  .2  25  20  Made with NZGrapher  Fuel Efficiency by Weight  Weight (kg)  www.mathsnz com | I am looking at buying a car and I have heard that heavy cars use more petrol1. Therefore I wonder if there is a relationship between the weight of a car and the car’s fuel efficiency for the purpose of predicting the fuel efficiency.    1. <http://www.theaa.com/motoring_advice/car-buyers-guide/cbg_emissions.html> |
| Machine generated alternative text: 320  280  240  E 220  Made with NZGrapher  2. Marathon Time by Stride Length  d:  140  1 go 200  mat hsnz com  150 160  170  Stride Length (cm) |  |
| Machine generated alternative text: 3. Rugby Players Weight by Height  Height (m)  110  100  Made with NZGrapher  w.w.'. mat hsnz com |  |
| Machine generated alternative text: 5000  4500  4000  6500  -4500  1 500  1000  Made with NZGrapher  4. Babies Birth Weight by Mothers Age  w.w.'. mat hsnz com  Mothers Age (years) |  |
| Machine generated alternative text: 50  30  .2  25  Made with NZGrapher  5. Fuel Efficiency by Engine Size  :o:  Engine Size (L)  www.mathsnz com |  |
| Machine generated alternative text: 12000  10000  02 0.3  Made with NZGrapher  0.4  6. Diamond Price by Size  0.5 0.6 0.7 0.8  Size (Carat)  www.mathsnz com |  |

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**Part 3.1: Identifying the Variables**

The next thing that we need to do is identify our variables and say what units are being used. The independent variable is the variable on the x-axis, and the dependent variable is the variable on the y-axis that we are wanting to predict.

Identify the variables for each of the graphs below. The first one has been done as an example for you.

|  |  |
| --- | --- |
| Machine generated alternative text: 40  E  35  .2  25  20  Made with NZGrapher  Fuel Efficiency by Weight  Weight (kg)  www.mathsnz com | The independent variable is the weight of the car, which is the mass of the car in kilograms. The dependent variable is the fuel efficiency in cities and on motorways, measured in kilometres per litre. |
| Machine generated alternative text: 320  280  240  E 220  Made with NZGrapher  2. Marathon Time by Stride Length  d:  140  1 go 200  mat hsnz com  150 160  170  Stride Length (cm) |  |
| Machine generated alternative text: 3. Rugby Players Weight by Height  Height (m)  110  100  Made with NZGrapher  w.w.'. mat hsnz com |  |
| Machine generated alternative text: 5000  4500  4000  6500  -4500  1 500  1000  Made with NZGrapher  4. Babies Birth Weight by Mothers Age  w.w.'. mat hsnz com  Mothers Age (years) |  |
| Machine generated alternative text: 50  30  .2  25  Made with NZGrapher  5. Fuel Efficiency by Engine Size  :o:  Engine Size (L)  www.mathsnz com |  |
| Machine generated alternative text: 12000  10000  02 0.3  Made with NZGrapher  0.4  6. Diamond Price by Size  0.5 0.6 0.7 0.8  Size (Carat)  www.mathsnz com |  |

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**Part 3.2: Naming the Source**

In order for our report to have validity, we need to state where the data has come from.

Name the source for each of the graphs below. The first one has been done as an example for you.

*Hint: Use the Information of all of the data sources.*

|  |  |
| --- | --- |
| Machine generated alternative text: 40  E  35  .2  25  20  Made with NZGrapher  Fuel Efficiency by Weight  Weight (kg)  www.mathsnz com | The data used in this investigation is from new vehicles sold in America in 1993. |
| Machine generated alternative text: 320  280  240  E 220  Made with NZGrapher  2. Marathon Time by Stride Length  d:  140  1 go 200  mat hsnz com  150 160  170  Stride Length (cm) |  |
| Machine generated alternative text: 3. Rugby Players Weight by Height  Height (m)  110  100  Made with NZGrapher  w.w.'. mat hsnz com |  |
| Machine generated alternative text: 5000  4500  4000  6500  -4500  1 500  1000  Made with NZGrapher  4. Babies Birth Weight by Mothers Age  w.w.'. mat hsnz com  Mothers Age (years) |  |
| Machine generated alternative text: 50  30  .2  25  Made with NZGrapher  5. Fuel Efficiency by Engine Size  :o:  Engine Size (L)  www.mathsnz com |  |
| Machine generated alternative text: 12000  10000  02 0.3  Made with NZGrapher  0.4  6. Diamond Price by Size  0.5 0.6 0.7 0.8  Size (Carat)  www.mathsnz com |  |

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**Part 4: Data – Using NZGrapher**

The next section that we need to do is the data section. This is reproducing the graphs on Page 2 using NZGrapher. The example below will go through using the cars dataset for weight by engine size.

NZGrapher runs on anything with a browser… Macs, PCs, iPad, Android, ChromeBooks and more.

|  |  |
| --- | --- |
| First up we need to start NZGrapher by going to the link in the box to the right. | [www.jake4maths.com/grapher](http://www.jake4maths.com/grapher) |
| The first time you load NZGrapher it will display an overlay with descriptions as to what all the different areas do as shown to the right. To load your data in either select it from the dropdown in the top right, or upload it in the top left corner and press go. | C:\CA825085\2CE2AD46-5EBC-49CC-9EA8-1E55C7ACFC2C_files\image015.png |
| To draw a scatter plot there are just three things you need to do.   1. Select the x-variable… this is your independent variable that will be on the x-axis, in this case it’s engine size. 2. Select the y-variable… this is your response or dependent variable, in this case it’s weight. 3. Select the graph type… for this we want the scatter.   This will give a graph with just the points. You need to check the graph title and axis labels to make sure they are appropriate (include units where necessary) and press update graph.  To save or copy the graph just right click on it and press ‘Copy Image’ or ‘Save Image As’ or whatever your device says that is similar.   1. Once you have the graph without any regression line you should add in the regression line by pressing the ‘Regression Line’ check box.   *Note 1: The summary statistics are automatically overlaid in red, if you want to remove them just un-tick the summary statistics box.*  *Note 2: If you want to identify the outliers, if you click the ‘Point Labels’ checkbox this will add little numbers next to the points that correspond with the point id.* | C:\CA825085\2CE2AD46-5EBC-49CC-9EA8-1E55C7ACFC2C_files\image016.png    C:\CA825085\2CE2AD46-5EBC-49CC-9EA8-1E55C7ACFC2C_files\image017.png |

Now it is your turn. For each dataset you need to produce the scatter plot for each dataset.

Don’t forget to add appropriate titles and units to your graph and axis.

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**Part 5: Analysis**

We now start on the Analysis section of our report. The acronym we use for this section is TARSOG.

The most important thing that you need to remember in this section is that what you can see with your eyes in the most important, not just the numbers, and your comments should be linked to the context.

Note: to be going for a **Merit** or **Excellence** Grade in this TARSOG section you need to be justifying these features in context and using research to back up your statements.

**Part 5.1: Trend**

The first comment we need to make is about the trend. There are three statements we need to make about the trend.

* How strong the trend is: weak, moderate, strong (or somewhere in between),
* If the trend is positive or negative (does it go up or down) and
* Is the trend linear (most circumstances we look at – forming a straight line) or non-linear.

Write a trend statement for each of the datasets. Again the first one has been done for you.

|  |  |
| --- | --- |
| Machine generated alternative text: 40  E  35  .2  25  20  Made with NZGrapher  Fuel Efficiency by Weight  Weight (kg)  www.mathsnz com | From the graph I can see a strong negative linear relationship between the weight of a car and the fuel efficiency of the car. |
| Machine generated alternative text: 320  280  240  E 220  Made with NZGrapher  2. Marathon Time by Stride Length  d:  140  1 go 200  mat hsnz com  150 160  170  Stride Length (cm) |  |
| Machine generated alternative text: 3. Rugby Players Weight by Height  Height (m)  110  100  Made with NZGrapher  w.w.'. mat hsnz com |  |
| Machine generated alternative text: 5000  4500  4000  6500  -4500  1 500  1000  Made with NZGrapher  4. Babies Birth Weight by Mothers Age  w.w.'. mat hsnz com  Mothers Age (years) |  |
| Machine generated alternative text: 50  30  .2  25  Made with NZGrapher  5. Fuel Efficiency by Engine Size  :o:  Engine Size (L)  www.mathsnz com |  |
| Machine generated alternative text: 12000  10000  02 0.3  Made with NZGrapher  0.4  6. Diamond Price by Size  0.5 0.6 0.7 0.8  Size (Carat)  www.mathsnz com |  |

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**Part 5.2: Association**

Association is about explaining why the relationship is either positive or negative, and it is important to link back to the context.

Discuss the association for each of the sets of data, the first one has been done for you

|  |  |
| --- | --- |
| Machine generated alternative text: 40  E  35  .2  25  20  Made with NZGrapher  Fuel Efficiency by Weight  Weight (kg)  www.mathsnz com | I can see that the association is negative because as the weight of the car increases, the fuel efficiency of the car decreases. |
| Machine generated alternative text: 320  280  240  E 220  Made with NZGrapher  2. Marathon Time by Stride Length  d:  140  1 go 200  mat hsnz com  150 160  170  Stride Length (cm) |  |
| Machine generated alternative text: 3. Rugby Players Weight by Height  Height (m)  110  100  Made with NZGrapher  w.w.'. mat hsnz com |  |
| Machine generated alternative text: 5000  4500  4000  6500  -4500  1 500  1000  Made with NZGrapher  4. Babies Birth Weight by Mothers Age  w.w.'. mat hsnz com  Mothers Age (years) |  |
| Machine generated alternative text: 50  30  .2  25  Made with NZGrapher  5. Fuel Efficiency by Engine Size  :o:  Engine Size (L)  www.mathsnz com |  |
| Machine generated alternative text: 12000  10000  02 0.3  Made with NZGrapher  0.4  6. Diamond Price by Size  0.5 0.6 0.7 0.8  Size (Carat)  www.mathsnz com |  |

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**Part 5.3: Relationship**

The statement about the relationship is about justifying the strength of the trend that **you can see** on your graph. It is important that you are commenting on what **you can see**. You can use the correlation coefficient (r-value) to **back up** your strength statement, but it should only be used as a backup… **what you can see** is the most important.

The r-value is a number between -1 and 1 indicating how strong the relationship is. The closer it is to 1 or -1 the stronger the relationship is, and the closer it is to zero the weaker the relationship is. A positive r-value indicates that the trend is positive, a negative r-value indicated the trend is negative.

The number line below is **just a guide**, remember what you can see with your eyes is most important.

Machine generated alternative text:
Strong Moderate 
Weak 
-0.4 
No Relationship 
Weak 
0.4 
Moderate 
Strong 
0.6 
-0.8 
-0.6 
-0.2 
o 
0.2 
0.8 

Discuss the relationship for each of the sets of data, the first one has been done for you.

|  |  |
| --- | --- |
| Machine generated alternative text: 40  E  35  .2  25  20  Made with NZGrapher  Fuel Efficiency by Weight  Weight (kg)  www.mathsnz com | The relationship is strong and linear as I can see most the points form a fairly consistent pattern. This is confirmed by the correlation coefficient of  -0.8431, indicating that the linear relationship is quite strong as r is between -0.75 and -1. |
| Machine generated alternative text: 320  280  240  E 220  Made with NZGrapher  2. Marathon Time by Stride Length  d:  140  1 go 200  mat hsnz com  150 160  170  Stride Length (cm) |  |
| Machine generated alternative text: 3. Rugby Players Weight by Height  Height (m)  110  100  Made with NZGrapher  w.w.'. mat hsnz com |  |
| Machine generated alternative text: 5000  4500  4000  6500  -4500  1 500  1000  Made with NZGrapher  4. Babies Birth Weight by Mothers Age  w.w.'. mat hsnz com  Mothers Age (years) |  |
| Machine generated alternative text: 50  30  .2  25  Made with NZGrapher  5. Fuel Efficiency by Engine Size  :o:  Engine Size (L)  www.mathsnz com |  |
| Machine generated alternative text: 12000  10000  02 0.3  Made with NZGrapher  0.4  6. Diamond Price by Size  0.5 0.6 0.7 0.8  Size (Carat)  www.mathsnz com |  |

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**Part 5.4: Scatter**

In the scatter section you need to look and see how consistent the scatter is. Are there any areas that are denser or sparser than others?

Discuss the scatter for each of the sets of data, the first one has been done for you.

|  |  |
| --- | --- |
| Machine generated alternative text: 40  E  35  .2  25  20  Made with NZGrapher  Fuel Efficiency by Weight  Weight (kg)  www.mathsnz com | The scatter appears to be reasonably consistent for car weights above about 1000 kg but below this there does appear to be fewer cars, probably due to not many small cars being manufactured. |
| Machine generated alternative text: 320  280  240  E 220  Made with NZGrapher  2. Marathon Time by Stride Length  d:  140  1 go 200  mat hsnz com  150 160  170  Stride Length (cm) |  |
| Machine generated alternative text: 3. Rugby Players Weight by Height  Height (m)  110  100  Made with NZGrapher  w.w.'. mat hsnz com |  |
| Machine generated alternative text: 5000  4500  4000  6500  -4500  1 500  1000  Made with NZGrapher  4. Babies Birth Weight by Mothers Age  w.w.'. mat hsnz com  Mothers Age (years) |  |
| Machine generated alternative text: 50  30  .2  25  Made with NZGrapher  5. Fuel Efficiency by Engine Size  :o:  Engine Size (L)  www.mathsnz com |  |
| Machine generated alternative text: 12000  10000  02 0.3  Made with NZGrapher  0.4  6. Diamond Price by Size  0.5 0.6 0.7 0.8  Size (Carat)  www.mathsnz com |  |

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**Part 5.5: Unusual Values (Outliers)**

In a large number of graphs there will be points (1 or 2) that do not follow the trend. These are called unusual values or outliers. When you identify an outlier you need to find it on the data list and find out as much information about it as you can in order to explain why it might be an outlier.

You could be thinking about the impact of these outliers on your model.

Discuss the outliers for each of the sets of data, the first one has been done for you.

|  |  |
| --- | --- |
| Machine generated alternative text: 40  E  35  .2  25  20  Made with NZGrapher  Fuel Efficiency by Weight  Weight (kg)  www.mathsnz com | There are two cars that have higher fuel efficiency rates than expected. The first is a Geo Metro with a weight of 769 kg and a fuel efficiency of 46 km/l in the city. The second is a Honda Civic with a weight of 1066 kg and a fuel efficiency of 42 km/l. Both of these cars have very small engines so I expect this will have increased their fuel efficiency. |
| Machine generated alternative text: 320  280  240  E 220  Made with NZGrapher  2. Marathon Time by Stride Length  d:  140  1 go 200  mat hsnz com  150 160  170  Stride Length (cm) |  |
| Machine generated alternative text: 3. Rugby Players Weight by Height  Height (m)  110  100  Made with NZGrapher  w.w.'. mat hsnz com |  |
| Machine generated alternative text: 5000  4500  4000  6500  -4500  1 500  1000  Made with NZGrapher  4. Babies Birth Weight by Mothers Age  w.w.'. mat hsnz com  Mothers Age (years) |  |
| Machine generated alternative text: 50  30  .2  25  Made with NZGrapher  5. Fuel Efficiency by Engine Size  :o:  Engine Size (L)  www.mathsnz com |  |
| Machine generated alternative text: 12000  10000  02 0.3  Made with NZGrapher  0.4  6. Diamond Price by Size  0.5 0.6 0.7 0.8  Size (Carat)  www.mathsnz com |  |

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**Part 5.6: Grouping**

Sometimes in graphs you can end up with two groups (or clusters) of data. If this happens you need to comment on it and what might be causing it, otherwise you can comment that there is not any obvious grouping. Again link it to what you can see.

Discuss the grouping for each of the sets of data, the first one has been done for you.

|  |  |
| --- | --- |
| Machine generated alternative text: 40  E  35  .2  25  20  Made with NZGrapher  Fuel Efficiency by Weight  Weight (kg)  www.mathsnz com | Looking at the graph I cannot see any obvious groupings. This is what I would expect as there are not really two different sizes of cars, they are all on a continuous range. |
| Machine generated alternative text: 320  280  240  E 220  Made with NZGrapher  2. Marathon Time by Stride Length  d:  140  1 go 200  mat hsnz com  150 160  170  Stride Length (cm) |  |
| Machine generated alternative text: 3. Rugby Players Weight by Height  Height (m)  110  100  Made with NZGrapher  w.w.'. mat hsnz com |  |
| Machine generated alternative text: 5000  4500  4000  6500  -4500  1 500  1000  Made with NZGrapher  4. Babies Birth Weight by Mothers Age  w.w.'. mat hsnz com  Mothers Age (years) |  |
| Machine generated alternative text: 50  30  .2  25  Made with NZGrapher  5. Fuel Efficiency by Engine Size  :o:  Engine Size (L)  www.mathsnz com |  |
| Machine generated alternative text: 12000  10000  02 0.3  Made with NZGrapher  0.4  6. Diamond Price by Size  0.5 0.6 0.7 0.8  Size (Carat)  www.mathsnz com |  |

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**Part 5.7: Interpretation of Regression Line**

*Teachers note: this is not a requirement of the standard, but it does round out the discussion nicely.*

One of the key bits of information that we get given from NZGrapher is the equation of the regression line. Interpreting the gradient of this regression line is an important comment to make. It is vital that you realise that this is only giving the **average** increase over the whole graph, and not a fixed amount for every unit.

Interpret the regression line for each of the sets of data, the first one has been done for you.

|  |  |
| --- | --- |
| Machine generated alternative text: 1 . Fuel Efficiency by Weight  Fuel Efficiency in City (km/l) -0.01770g•weight (kg) + 47.048  r: -084311  num  40  E  35  30  .2  25  20  Made with NZGrapher  a::  Weight (kg)  www.mathsnz com | The regression line of  Fuel Efficiency = -0.017709 × Weight + 47.048 means that for every one kilogram increase in the car’s weight, the fuel efficiency decreases by 0.017709 kilometres per litre on average. |
| Machine generated alternative text: 320  280  240  E 220  Made with NZGrapher  2. Marathon Time by Stride Length  Tige Taken (minutes) -1.86g6*Stride length (cm) + 530.33  r -0.90507  num goo  1 go 200  mat hsnz com  140 150 160  Stride length (cm)  170 |  |
| Machine generated alternative text: 3. Rugby Players Weight by Height  weight (kg) 66.726*Height (m) + -21866  w.w.'. mat hsnz com  r 043443  num 135  110  100  Made with NZGrapher  Height (m) |  |
| Machine generated alternative text: 5000  4500  4000  6500  -4500  1 500  1000  4. Babies Birth Weight by Mothers Age  Birth Weight (g) 12.364*Mothers Age (years) + 2657.3  r 008987  num 189  w.w.'. mat hsnz com  Mothers Age (years)  Made with NZGrapher |  |
| Machine generated alternative text: 50  30  .2  25  5. Fuel Efficiency by Engine Size  Fuel Efficiency on Open Road (km/l) -3.221 "Engine Size (L) + 37.680  r -0.62679  num 93  Engine Size (L)  www.mathsnz com  Made with NZGrapher |  |
| Machine generated alternative text: 12000  10000  6. Diamond Price by Size  Price (IJSD) 7788.7*Size (Carat) + -1450.0  r z 0.94701  num 236  www.mathsnz com  0.4  02 0.3  Made with NZGrapher  0.5 0.6 0.7 0.8  Size (Carat) |  |

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**Part 5.8: Predictions**

As well as interpreting the regression line we need to use this line to make **at least two** predictions and comment on how reliable we think the predictions are based on the strength of the relationship and the scatter on the graph close to the point we are predicting.

In order to do this you need to substitute two different values into the equation. With bivariate data we can only safely make predictions inside our data range, so your predictions should be able to be plotted on your graph. It is also vital that you **round the prediction sensibly** (usually the same as the original data for that variable was rounded to).

You could expand on this further by discussing the confidence in the predictions in depth or linking this to the residuals (see section later on). You could also reflect on how relevant these predictions are.

Make two predictions inside the data range for each of the sets of data, the first one has been done for you.

|  |  |
| --- | --- |
| Machine generated alternative text: 1 . Fuel Efficiency by Weight  Fuel Efficiency in City (km/l) -0.01770g•weight (kg) + 47.048  r: -084311  num  40  E  35  30  .2  25  20  Made with NZGrapher  a::  Weight (kg)  www.mathsnz com | *-0.017709 × 1200 + 47.048 = 26.7972*  Based on my regression line I would predict that a car that weighs 1200 kg would have a fuel efficiency of approximately 27 kilometres per litre. I am / am not confident in this prediction because...    *-0.017709 × 1600 + 47.048 = 18.7136*  Based on my regression line I would predict that a car that weighs 1600 kg would have a fuel efficiency of approximately 19 kilometres per litre. I am / am not confident in this prediction because... |
| Machine generated alternative text: 320  280  240  E 220  Made with NZGrapher  2. Marathon Time by Stride Length  Tige Taken (minutes) -1.86g6*Stride length (cm) + 530.33  r -0.90507  num goo  1 go 200  mat hsnz com  140 150 160  Stride length (cm)  170 |  |
| Machine generated alternative text: 3. Rugby Players Weight by Height  weight (kg) 66.726*Height (m) + -21866  w.w.'. mat hsnz com  r 043443  num 135  110  100  Made with NZGrapher  Height (m) |  |
| Machine generated alternative text: 5000  4500  4000  6500  -4500  1 500  1000  4. Babies Birth Weight by Mothers Age  Birth Weight (g) 12.364*Mothers Age (years) + 2657.3  r 008987  num 189  w.w.'. mat hsnz com  Mothers Age (years)  Made with NZGrapher |  |
| Machine generated alternative text: 50  30  .2  25  5. Fuel Efficiency by Engine Size  Fuel Efficiency on Open Road (km/l) -3.221 "Engine Size (L) + 37.680  r -0.62679  num 93  Engine Size (L)  www.mathsnz com  Made with NZGrapher |  |
| Machine generated alternative text: 12000  10000  6. Diamond Price by Size  Price (IJSD) 7788.7*Size (Carat) + -1450.0  r z 0.94701  num 236  www.mathsnz com  0.4  02 0.3  Made with NZGrapher  0.5 0.6 0.7 0.8  Size (Carat) |  |

Machine generated alternative text:
Maths 
stvcdewts 
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**Part 5.9: Using the Graph for Confidence in Predictions**

Machine generated alternative text:
50 
30 
25 
15 
10 
1. Cars fuel efficiency by weight 
Fuel Efficiency (km/D : -0.01770g•weight (kg) + 47.048 
F -0.8431 1 
num : 93 
0 00 
co 00 
200 ( 
1000 
1400 
1600 
oco o 
1800 
1200 
Weight (kg) 
Made With NZGrapher 
wwrw.mathsnz.com 

Looking at graph and how the scatter is

can give us a good indication as to how

reliable our predictions are likely to be,

and therefore create an interval that we

think our predictions might be in.

If we look at this graph here we can see

that most of the points are between the

dotted lines (you put these in manually

by eye).

If we go back to the prediction that I made

earlier... we can’t be completely certain a

car that weighs 1200kg will have a fuel

efficiency of 27 km/l, but we can be

reasonably confident that the fuel

efficiency for that car will be somewhere between 20 and 31 km/l.

**Part 5.10: Cause and Effect and Correlation**

*Teachers note: this is not a requirement of the standard, but it does round out the discussion nicely.*

While there is a relationship between two variables, there are two reasons why we cannot make causal statements:

-        We don’t know the direction of the cause – Does X cause Y or does Y cause X?

-        A third variable may be involved that is responsive for the covariance between X and Y, we call this a lurking variable.

Causal relationships can only be determined by controlled experiments, which we look at in a different standard.

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**Part 5.11: Residuals**

*Teachers note: this is not a requirement of the standard, but it does round out the discussion nicely.*

One of the ways that we can analyse how well our model fits the data and therefore how reliable our predictions are is by looking at the residuals. We can create a plot of the residuals using NZGrapher.

Machine generated alternative text:
l.Carsfuelefficiencybyweight 

This part is really easy… all you need to do is

change the graph type from the graph that we

did earlier to ‘residuals’.

This gives the output shown to the right, which

shows the expected (or fitted) values on the

x-axis and the difference between the fitted

and the actual (the residual) on the y-axis.

The line that is put in is a weighted average

curve that shows the overall trend of the data.

For each of the sets of data, generate the residuals plot and use this to justify how accurate you think your predictions are, the first one has been done for you.

|  |  |
| --- | --- |
| Machine generated alternative text: 3  2  Fuel Efficiency by Weight  32 34  '.•.wv.'. mat hsnz com  28  30  22  18  20  24  Fitted  26  Made with NZGrapher | Looking at my residuals plot I think that the first car I predicted to have a fuel efficiency of 27 km/l might actually be slightly less than this due to most of the values being below the predicted line in the middle of the range. Based on looking at the residuals the car that I predicted to have a fuel efficiency of 19 km/l probably will be very close to this as all of the points round 19 km/l are very close to the predicted line. |
| Machine generated alternative text: -20  160 180  Made with NZGrapher  200  2. Marathon Time by Stride Length  220  240 260  Fitted  280  300  320 340  www.mathsnz com |  |
| Machine generated alternative text: 10  Made with NZGrapher  3. Rugby Players Weight by Height  115  www.mathsnz com  105  Fitted  110 |  |
| Machine generated alternative text: 2800 2850  Made with NZGrapher  4. Babies Birth Weight by Mothers Age  2900  2950  3000 3050  Fitted  3100  3150  3200 3250  mat hsnz com |  |
| Machine generated alternative text: 3  5. Fuel Efficiency by Engine Size  2  Fitted  www.mathsnz com  Made with NZGrapher |  |
| Machine generated alternative text: 3  -1000  Made with NZGrapher  1000  6. Diamond Price by Size  3000 4000  Fitted  6000  7000  8000  mat hsnz com  2000  5000 |  |

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**Part 6: Conclusion**

We now need to make a concluding statement to summarise our report. You need to include a statement around the relationship, and it needs to be linked back to what you are investigating.

Make a conclusion for each of the sets of data, the first one has been done for you.

|  |  |
| --- | --- |
| Machine generated alternative text: 1 . Fuel Efficiency by Weight  Fuel Efficiency in City (km/l) -0.01770g•weight (kg) + 47.048  r: -084311  num  40  E  35  30  .2  25  20  Made with NZGrapher  a::  Weight (kg)  www.mathsnz com | In conclusion I think there is a strong negative relationship between the weight of cars and the fuel efficiency – the heaver the car, the more fuel it will use, therefore if we know the weight of a car we should be able to predict the fuel efficiency. This is useful for me to know because if I want a car that will use less petrol I know I should buy a lighter car. |
| Machine generated alternative text: 320  280  240  E 220  Made with NZGrapher  2. Marathon Time by Stride Length  Tige Taken (minutes) -1.86g6*Stride length (cm) + 530.33  r -0.90507  num goo  1 go 200  mat hsnz com  140 150 160  Stride length (cm)  170 |  |
| Machine generated alternative text: 3. Rugby Players Weight by Height  weight (kg) 66.726*Height (m) + -21866  w.w.'. mat hsnz com  r 043443  num 135  110  100  Made with NZGrapher  Height (m) |  |
| Machine generated alternative text: 5000  4500  4000  6500  -4500  1 500  1000  4. Babies Birth Weight by Mothers Age  Birth Weight (g) 12.364*Mothers Age (years) + 2657.3  r 008987  num 189  w.w.'. mat hsnz com  Mothers Age (years)  Made with NZGrapher |  |
| Machine generated alternative text: 50  30  .2  25  5. Fuel Efficiency by Engine Size  Fuel Efficiency on Open Road (km/l) -3.221 "Engine Size (L) + 37.680  r -0.62679  num 93  Engine Size (L)  www.mathsnz com  Made with NZGrapher |  |
| Machine generated alternative text: 12000  10000  6. Diamond Price by Size  Price (IJSD) 7788.7*Size (Carat) + -1450.0  r z 0.94701  num 236  www.mathsnz com  0.4  02 0.3  Made with NZGrapher  0.5 0.6 0.7 0.8  Size (Carat) |  |

Congratulations, you now have written up a report for 5 different sets of data.

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**Part 7a: Writing Your Own Internal 1**

Using the framework below write a report on the kiwi data.

**Kiwi Birds**

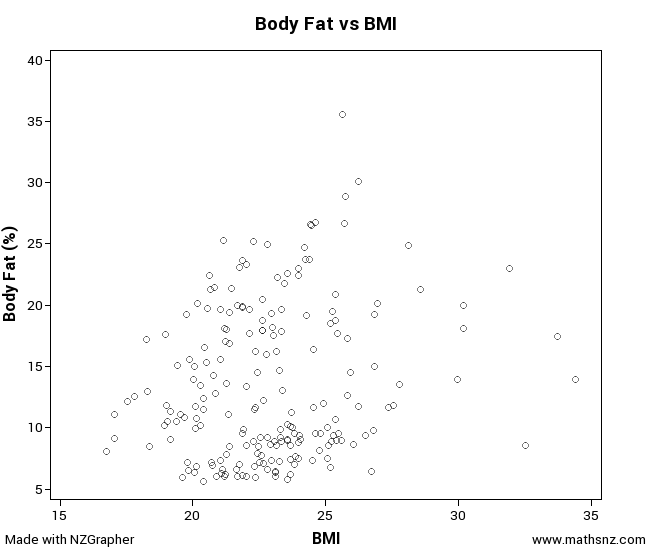
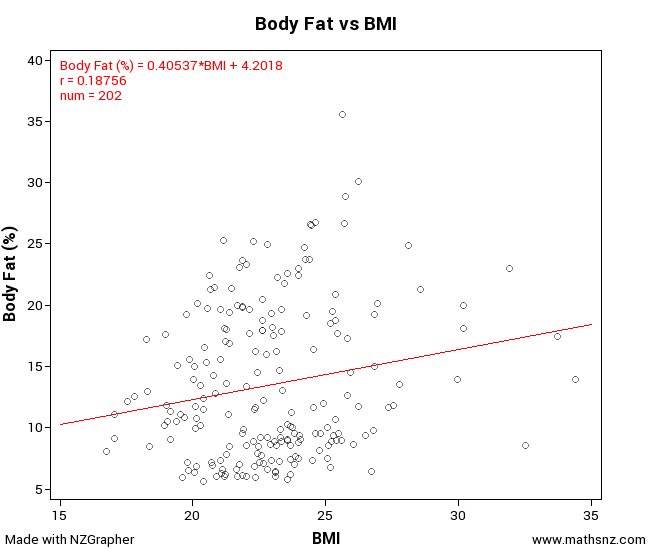
|  |  |
| --- | --- |
| **Problem** |  |
|  | Question and Purpose linked to context, reason for investigation given. |
| **Plan** |  |
|  | Variables Identified |
|  | Source Named |
| **Data** |  |
| C:\CA825085\2CE2AD46-5EBC-49CC-9EA8-1E55C7ACFC2C_files\image027.png | Graph without Regression Line Given |
| **Analysis** |  |
|  | Trend |
|  | Association |
|  | Relationship |
|  | Scatter |
|  | Outliers |
|  | Grouping |
| C:\CA825085\2CE2AD46-5EBC-49CC-9EA8-1E55C7ACFC2C_files\image028.png | Graph with Regression line given |
|  | Interpretation of Regression Line |
|  | Predictions |
| **Conclusion** |  |
|  | Summarise and link back to the purpose |

Machine generated alternative text:
Maths 
stvcdewts 
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**Part 7b: Writing Your Own Internal 1**

This time you have just been provided with a title and graphs. Using these write your own internal. This is using the Sports Science dataset.

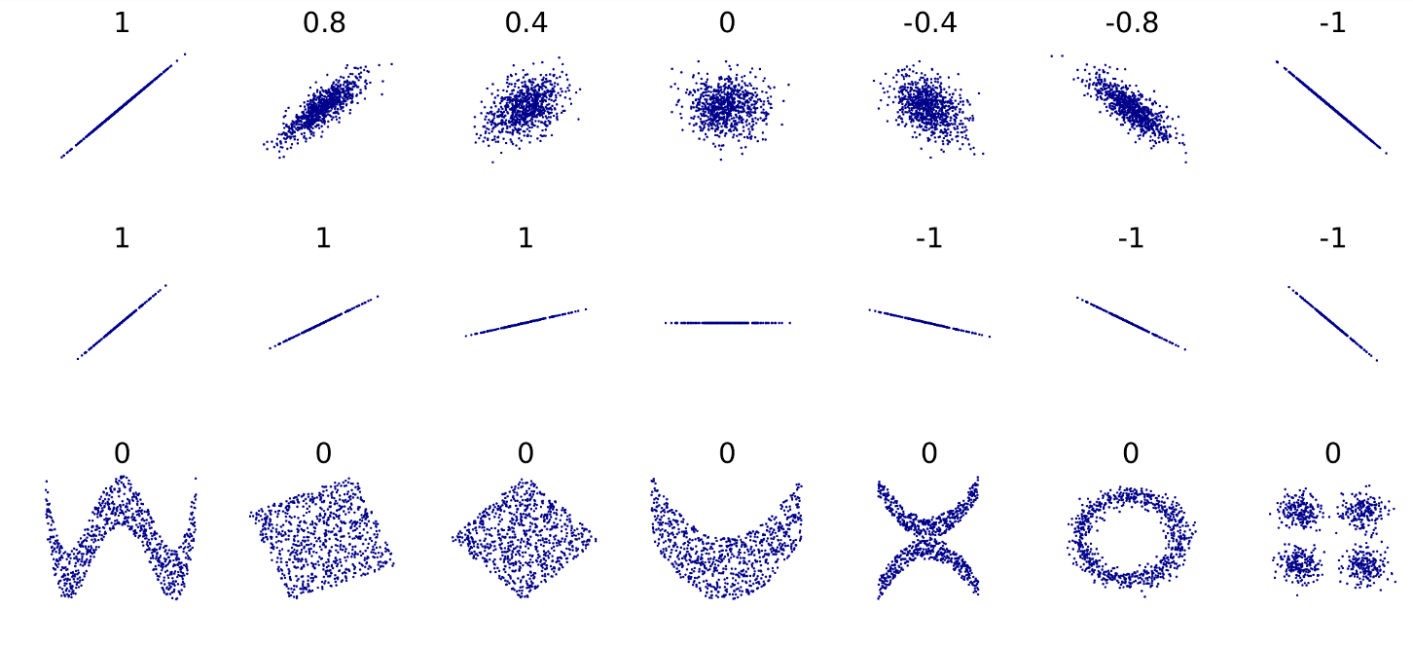
**BMI and Body Fat**

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Machine generated alternative text:
Maths 
stvcdewts 
NCEA Level 3 - 3.9 Bivariate Data 
students.mathsnz.com 

**Part 8: Appendix A – Examples of Correlation Coefficient (r)**

Below are graphs showing the different correlation coefficients (r values) above each one.



Source: <http://upload.wikimedia.org/wikipedia/commons/d/d4/Correlation_examples2.svg>

**Part 8: Appendix B – Separating Variables**

*Teachers note: this is not a requirement of the standard, but it does round out the discussion nicely.*

Machine generated alternative text:
1. Сагз е'Нс'епсу Ьу 

One of the things you can do to round

out the discussion is separate out the

data by a categorical variable to see

if this is affecting the trend. You do this

by choosing a variable in the ‘Subset’

section on NZGrapher. This gives the

output on the right.

Machine generated alternative text:
Maths 
stvcdewts 
NCEA Level 3 - 3.9 Bivariate Data 
students.mathsnz.com 

**Part 8: Appendix C – Data Set Information**

**Babies**

The data on 189 births were collected at Baystate Medical Center, Springfield, Mass. during 1986.

The goal of this study was to identify risk factors associated with giving birth to a low birth weight baby (weighing less than 2500 grams). Data was collected on 189 women, 59 of which had low birth weight babies and 130 of which had normal birth weight babies.

|  |  |
| --- | --- |
| **Variable** | **Description** |
| LowBirthWeight | No = Birth Weight >= 2500g  Yes = Birth Weight < 2500g |
| MothersAge | Age of the Mother in Years |
| Race | Race of the mother |
| MotherSmoke | Smoking Status During Pregnancy |
| FTV | Number of Physician Visits During the First Trimester |
| BirthWeight | Birth Weight in Grams |

**Cars**

With rising costs of owning and running a car, and environmental awareness, buyers are becoming more conscious of the features when purchasing new cars. The data supplied is for new vehicles sold in America in 1993.

|  |  |
| --- | --- |
| **Variable** | **Description** |
| Vehicle Name |  |
| Origin | Country of manufacture   * + America   + Foreign |
| Price | US $1000 |
| Type | Small, midsize, large, compact, sporty, van |
| City | Fuel efficiency in kilometres per litre in cities and on motorways |
| OpenRoad | Fuel efficiency in kilometres per litre on country and open roads |
| Drive Train | Front Wheel Drive  Rear Wheel Drive |
| Engine Size | Size in litres |
| Manual Transmission | Yes  No |
| Weight | Weight of car in Kg |

**Diamonds**

Every diamond is unique, and there are a variety of factors which affect the price of a diamond. Insurance companies in particular are concerned that stones are valued correctly.

Data on 308 round diamond stones was collected from a Singapore based retailer of diamond jewellery, who had the stones valued.

|  |  |
| --- | --- |
| **Variable** | **Description** |
| Carat | Weight of diamond stones in carat units 1 carat = 0.2 grams |
| Colour | Numerical value given for quality of colour ranging from 1=colourless to 6=near colourless |
| Clarity | Average = score 1, 2 or 3  Above average = score 4, 5 or 6 |
| Lab | Laboratory that tested & valued the diamond  1 = laboratory 1  2 = laboratory 2 |
| Price | Price in US dollars |

**Kiwi**

A sample of kiwi birds around New Zealand was collected in order to help with conservation efforts. The original data is from: <http://www.kiwisforkiwi.org/> and was sourced from the secondary school guides *(*[*http://seniorsecondary.tki.org.nz/Mathematics-and-statistics/Achievement-objectives/AOs-by-level/AO-S7-1*](http://seniorsecondary.tki.org.nz/Mathematics-and-statistics/Achievement-objectives/AOs-by-level/AO-S7-1)*)*

|  |  |  |
| --- | --- | --- |
| **Variable** | **Description** |  |
| Species | GS-Great Spotted  NIBr-NorthIsland Brown  Tok-Southern Tokoeka |  |
| Gender | M-Male  F-Female |  |
| Weight(kg) | The weight of the kiwi bird in kg |  |
| Height(cm) | The height of the kiwi bird in cm |  |
| Location | NWN-North West Nelson  CW-Central Westland  EC-Eastern Canterbury  StI-Stewart Island  NF-North Fiordland | SF-South Fiordland  N-Northland  E-East North Island  W-West North Island |

*Teachers note: this is a synthesised dataset based on real data. At the time of creating the data set there were around 25,000 brown, 17,000 great spotted and 34,500 southern tokoeka. These numbers formed the basis of the data set, but instead of being out of around 76,000 the data set contains around 700 birds.*

*The data was generated using the population parameters, including gender, location, height, weight and species in Fathom. The size of the population was so that it was too big to use all the data (when doing by hand) but not too big that it couldn’t be created for students to use as a “population” to sample from.*

**Marathon**

The data is a sample taken from marathons in NZ.

It is a simple random sample of 200 athletes.

|  |  |
| --- | --- |
| **Variable** | **Description** |
| Minutes | How many minutes they completed the marathon in |
| Gender | Male (M) or Female (F) |
| AgeGroup | Younger (under 40) or older (over 40) |
| StridelengthCM | The persons average stride length over the marathon in cm. |

**Rugby**

The data is real data and comes from <http://www.rugby-sidestep-central.com/>

|  |  |
| --- | --- |
| **Variable** | **Description** |
| Country | New Zealand or South Africa |
| Position | Forward or Back |
| Weight | The weight of the player in kilograms (kg) |
| Height | The height of the player in metres (m) |

Machine generated alternative text:
Maths 
stvcdewts 
NCEA Level 3 - 3.9 Bivariate Data 
students.mathsnz.com 

**Part 9: Assessment Guidelines – 91581 – Investigate Bivariate Measurement Data**

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Achieved (all compulsory)** | **Merit** | **Excellence** |
| **Problem** | An appropriate relationship question is posed in context and linked to research.  Statement given about why variables were chosen in context. | The question is justified in context and linked to research. | The choice of variables is reflected on and linked to the context and research. |
| **Plan** | Data source is identified.  The explanatory and response variables are clear. |  |  |
| **Data** | Scatter plot(s) is produced with tile and labelled axis and regression line fitted. | Residuals plot may be produced. |  |
| **Analysis** | Features in the data are identified from a visual inspection and described. This should include:   * + Trend (linear or not)   + Association (direction)   + Relationship (strength)   + Scatter   + Grouping   + Outliers   Other features and unusual points have been identified. | Findings are justified with reference to evidence from the displays and statistics and the links findings to their research and purpose.    Causation may be discussed in context.    The appropriateness of the model may justified by discussion of fit throughout the range of x-values in the data or the number of data points. An analysis of the residuals may be used. | Contextual evidence and research is integrated to support discussion about the features of the data.    Features are reflected on by discussing their relevance.    Improvements to the model may be considered by considering other variables (eg: separating the data into relevant subsets or looking at another related variable).    The adequacy and strength of the model is reflected upon.    A deeper understanding of the model is shown. |
| **Predictions** | A prediction is made in context that is sensible with respect to the context and uses units and sensible rounding. | The precision of the prediction could be discussed by reviewing the strength of the relationship and the scatter on the graph close to the relevant explanatory data value. | The choice of variable used for predictions is justified by giving reasons for using the selected one rather than others.    Reflection is made on predictions by discussing their relevance. |
| **Conclusion** | A conclusion is given that is consistent with the question and linked to the purpose. | The conclusion is linked to the question with contextual support. | The conclusion shows a deeper understanding of the data and research and contextual reasons are made to support findings. |

Final grades will be decided using professional judgement based on a holistic examination of the evidence provided against the criteria in the Achievement Standard.